On the other hand, by taking account of the inertial term $\varkappa V_t$ in Eq. (3.5) problem (3.5)-(3.11) can be solved for arbitrary initial data. For small $\varkappa$, however, information on the initial velocity field, but not on the initial film profile, is rapidly forgotten in the motion process.

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# TURBULENT VISCOSITY FOR INCOMPRESSIBLE GRADIENT FLOWS BEFORE SEPARATION AND ON A ROUGH SURFACE

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The existing finite-difference methods of turbulent boundary-layer calculation, where various modifications of turbulent viscosity (mixing path lengths) are used for closure of the system of equations, lead to great differences between the calculated and experimental data for highly nonequilibrium (close to separation) flows [1-3]. One of the probable reasons for the observed disagreement is that existing models of turbulent viscosity contain insufficient information about the previous history of the flow. In particular, the relation for turbulent viscosity in the external part of a boundary layer [2] or, for instance, the relationship used in [4]

$$\mu_т = \rho(\lambda y_e)^2|\partial u/\partial y| \tag{1}$$

in explicit form is quite independent of the previous history. The value of $\lambda$ in (1) is constant and is usually taken as 0.09.

Correlation of the results of the experiments of Goldberg [3] and Schubauer and Spandenberg [1] showed that in the external part of the boundary layer the numerical value of $\lambda$ can vary approximately from 0.045 to 0.090, i.e., $\lambda \neq \text{const}$ along the streamline. At the same time, as will be shown below, the value of $\lambda$ can have a great effect on the fullness of the profile and the integral characteristics of the layer.

To determine the characteristics of the boundary layer before separation the authors of [4, 5] assumed various forms of dimensionless "universal" velocity profiles and obtained agreement with experiments by the introduction of empirical coefficients into the velocity profile relations.

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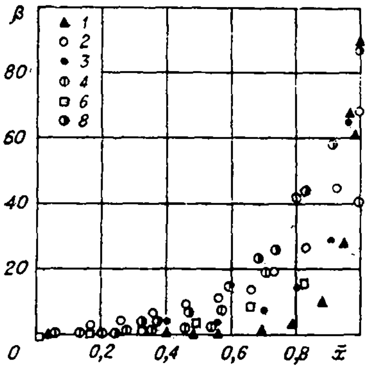


Fig. 1

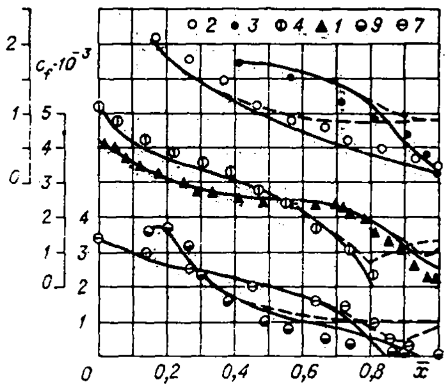


Fig. 2

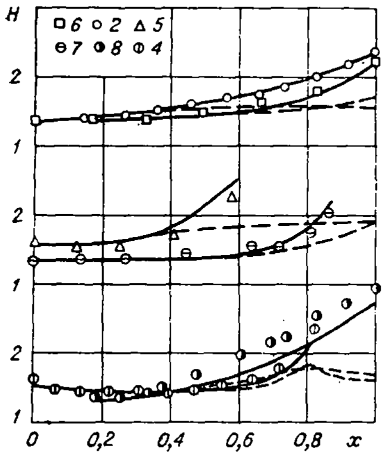


Fig. 3

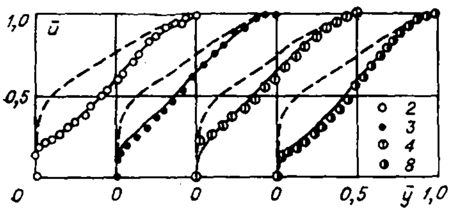


Fig. 4

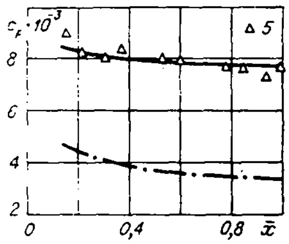


Fig. 5

In the present work we analyzed a large number of experimental works by Perry (2900), Schubauer and Klebanoff (2100), Newman (3500), Moses (3700, 3800), Schubauer and Spandenberg (4500, 4800), and Fraser (5000, 5100), dealing with flows with a strong pressure gradient, leading in several cases to boundary-layer separation (the figures in parentheses after the experimenters' names correspond to the numbers of cited experiments in [1]). This analysis revealed some characteristic features of flows before separation. First, the gradient of the external flow velocity ($du_\delta/dx$) on approach to separation tends to zero. The result of this is that the computational methods with the usual relations for closure of the turbulent boundary layer equations probably do not respond to the approach of separation and give the layer characteristics corresponding to flow on a plate. Secondly, flow before separation becomes significantly nonequilibrium and is accompanied by a sharp increase (Fig. 1) in the Clauser equilibrium parameter

$$\beta = (\delta_1/\tau_w)dp/dx,$$

where $\delta_1$ is the displacement thickness, $\tau_\omega$ is the friction stress on the wall, $dp/dx$ is the pressure gradient, x is the longitudinal coordinate, and $\bar{x} = x/x_{max}$.

In Figs. 1-4 the numbering of the experimental points (1-9) corresponds to the numbers of the experiments cited in [1]: 1) 2500; 2) 2900; 3) 3500; 4) 3700; 5) 3800; 6) 4500; 7) 4800; 8) 5000; 9) 5100.

In the light of these facts we can assume that instead of $\lambda$ in (1) it would be better to use a relation of the form $\lambda = \lambda(\mathrm{Re}_2, \delta/R, \zeta d\beta/dx)$, where $\mathrm{Re}_2$ is the Reynolds number related to the momentum thickness, $\delta$ is the boundary layer thickness, R is the instantaneous radius of the body, and $\zeta$ is the characteristic length. To verify this assumption on the basis of [6] we compiled a program for finite-difference computation of the turbulent boundary layer, which requires several comments. The finite-difference equation was constructed, as in [6], by integration over the control volume – a method previously proposed in [7].

For computation of the boundary layer on rough surfaces the relation for the total viscosity near the wall [6] generalized by the procedure in [8], where the roughness points were regarded as "vortex generators," and had the form

$$\mu_\Sigma = \mu + \rho k^2 y^2 \{1 - \exp[-y\sqrt{\tau\rho}/(\mu A)] + \exp[-A_r y\sqrt{\tau/\tau_w}/(Ah)]\}^2 \left|\frac{\partial u}{\partial y}\right|, \tag{3}$$

where $\mu$ is the dynamic viscosity, $\rho$ is the density, k = 0.40, y is the distance from the wall along a normal to the surface, $\tau$ is the tangential stress across the boundary layer, h is the height of the roughness points, u is the longitudinal velocity component, and $A_r$ depends on the shape and position of the roughness points. The relation for A was chosen on the basis of [9] in the form

$$A = 13.6 + 12.4\exp[-10.75(\rho v)_w/(\rho v_*)], \tag{4}$$

where $(\rho v)_w$ is the mass flux on the permeable wall; $v_* = \sqrt{\tau_w/\rho}$. Control calculations of the experiments of [9] for maximum injection and suction with the use of (4), carried out in accordance with the given program, show showed good agreement with the results of experiments. In [6] A was taken as 26, which leads to considerable disagreement of the calculated and experimental data [9].

In [6] the boundary condition for the finite-difference equation and the local friction coefficient $c_f$ were determined by using the approximation relations obtained by its authors on the basis of parametric calculations of Couette flow near the wall. In [6] these relations were extended to the "logarithmic" part of the velocity profile, which in several cases did not allow calculation of flows with strong pressure gradients and suction. The accuracy of the calculation was increased and these difficulties were overcome by the introduction of an iterative computation process, and the finite-difference equation was solved along with the ordinary differential equation

$$\frac{du_+}{dy_+} = \frac{2\tau_+}{1 + \sqrt{1 + 4y_+^2\tau_+\{1 - \exp(-y_+\sqrt{\tau_+}/A_+) + \exp[-A_r y_+\sqrt{\tau_+}/(Ah_+)]\}^2}}, \tag{5}$$

where

$$\begin{gathered} \tau_+ = (1 + p_+y_+ + m_+u_+)/(1 + y_+\cos\alpha/R_+); \\ u_+ = ku/v_*; \quad y_+ = ky\rho v_*/\mu; \quad p_+ = \mu(dp/dx)/(k\rho^2 v_*^3); \\ m_+ = m_w/(k\rho v_*); \; R_+ = y_+|_{y=R}; \; h_+ = y_+|_{y=h}; \; A_+ = kA \end{gathered} \tag{6}$$

($\alpha$ is the local angle of inclination of the surface to the axis of symmetry). Equation (4) was extended to the dimensionless distance $y_+ = 1.4$, i.e., was valid only in the immediate vicinity of the wall. Relation (6) was obtained by expansion of the tangential stress in the vicinity of the wall in a MacLaurin series using the equation of motion and the no-slip conditions. On the basis of numerical calculations by the expounded method and comparison with experimental data [1] for the plane and axisymmetric cases in which flows approaching separation were obtained, we established a relation for coefficient $\lambda$ in relation (1) in the form

$$\lambda = 0.09/(1 + cF), \tag{7}$$

where $c = 0.011\sqrt{\mathrm{Re}_2}/(1 + \delta\cos\alpha/R)^2$; $F = \delta_2|d\beta/dx|$ is a form parameter characterizing the degree of nonequilibrium of the flow; $\delta_2$ is the momentum thickness.

Formula (7) is valid for $\lambda \geq 0.045$. In the case where the function on the right-hand side of (7) becomes less than 0.045, we assumed that $\lambda = 0.045$. Thus, the relation for turbulent viscosity in the external part of the boundary layer can be put in the form

$$\mu_T = \rho[0.09y_l/(1 + cF)]^2|\partial u/\partial y|, \tag{8}$$

where $y_l$ is the distance from the wall, on which $u/u_\delta = 0.995$.

Figures 2-4 show the results of comparison of the calculated and experimental data for the friction coefficients, form parameters, and dimensionless velocity profiles, where the solid line corresponds to calculation using $\mu_T$, determined from relation (8), and the dashed line is obtained from (1) with $\lambda = 0.09$, $H = \delta_1/\delta_2$, $\bar{u} = u/u_\delta$, and $\bar{y} = y/\delta$. The velocity profiles in Fig. 4 correspond to the longitudinal coordinate $\bar{x} \approx 1$.

An analysis of the presented data indicates several characteristic features of computation of the turbulent boundary layer, where for closure of the equations we used relation (1) with $\lambda = 0.09$ and $\lambda$ according to (7). First of all, it should be noted that the use of $\lambda = 0.09$ did not give the "theoretical" separation realized in several experiments with which comparison was made. In the whole preseparation region of flow the calculation led to systematic overestimate of $c_f$, a great underestimate of the form parameter H, and significant filling of the velocity profiles in comparison with those observed in experiments. The value of $\beta$ in the calculations was practically constant. Thus, the calculated flow in the boundary layer becomes quasiequilibrium and does not correspond to the physics of the effect.

At the same time the use of relation (8) enabled us to obtain separation whose coordinate agreed satisfactorily with the experimental data. In the program the separation was fixed when the calculated value $c_f \to 0$. For the regions approaching separation the numerical values of $c_f$, H, and the fullness of the velocity profile in coordinates $\bar{u} = \bar{u}(\bar{y})$ corresponded fairly well with the corresponding experimental characteristics of the boundary layer. As in the experiments, there was a sharp increase in the equilibrium parameter $\beta$.

Thus, the more rapid (in comparison with the generally accepted) reduction of the turbulent viscosity along the streamline, corresponding to relation (8), which takes into account the degree of equilibrium of the flow, leads to a great improvement in the agreement of the calculated and experimental data. Hence, relation (8) probably allows a more satisfactory description of turbulent transfer in the external part of the boundary layer for regions close to separation.

The possibility of numerical calculation of the turbulent boundary layer on rough surfaces was confirmed by the use of relation (3). As an example we considered flow on a rough wire with aspect ratio $x_{max}/2R = 1740$, $R = 1$ mm, $h = 0.085$ mm at an air velocity $u\delta = 35$ m/sec. The value of $A_r$ in the turbulent viscosity formula (3) was taken as 26. Figure 5 shows the results of comparison of the calculated values of the mean friction coefficient $c_F$ (the solid line corresponds to flow on the wire and the dashed-dot line, to a rough plate) with experimental data [10]. It is apparent that the agreement of the theoretical and experimental values is satisfactory. Hence, the approach of [6], where the roughness points are regarded as "vortex generators," can be applied to the finite-difference method of calculation of turbulent boundary layer characteristics.

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